Part II. ch.1 Indifinite integration.

The Differential:

→ Differential are infinitely small quantities. We usually write differentials as dx, dy, dt and so on, where:

dx : is an infinitely small change in x,
dy : " " " " " " " " " y, and
dt : " " " " " " " " " t.

→ When comparing small changes in quantities that related to each other (like in case where y is some function of x), we say the differential dy of $y=f(x)$ is written:

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$$dy = f'(x)\cdot dx$$

→ Earlier in differentiation chapter, we wrote $\frac{dy}{dx}$ and $f'(x)$ to mean the same thing. We use $\frac{d}{dx}$ as an operator. We now see a different way to write and to think about the derivative.

→ Now, we treat $\frac{dy}{dx}$ more like a fraction rather than an operator.

Ex: Find the differential of (i) $y=3x^5-x$. (ii) $y=5x^2-4x+2$.

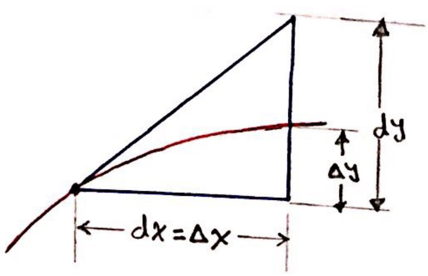
Ans: (i) $y=3x^5-x \Rightarrow \boxed{dy=(15x^4-1)\,dx}$.
(ii) $y=5x^2-4x+2 \Rightarrow \boxed{dy=(10x-4)\,dx}$.

To find the differential (dy), we just find the derivative and write it with (dx) on the right

Q: How are dy, dx and Δy and Δx related?

* Δy means "change in y" and Δx means "change in x".
* As Δx gets smaller, the ratio $\frac{\Delta y}{\Delta x}$ becomes closer to instantanous ratio $\frac{dy}{dx}$, i.e.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$



* Now, we go to see how differentials is used to perform the opposite process of differentiation, which first we'll call "anti-diff-" and later "integration"

Antiderivatives and the indifinite integral.

If we know that: $\frac{dy}{dx} = 3x^2$, what would I have to differentiate to get this result? [Think] ---mmm,

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$y = x^3$ is one antiderivative of $\frac{dy}{dx} = 3x^2$,

There are infinitely many other antiderivatives which also work

for example: $y = x^3 + 4$,
$y = x^3 + \pi$,
$y = x^3 + 43.4, \ldots$ } called antiderivatives of $\frac{dy}{dx} = 3x^2$

In general, we say $y = x^3 + C$ is the indifinite integral of $(3x^2)$, The number C is called constant of integration, Then

The indifinte integration is a family of functions [antiderivatives]

due to existance of the Constant [C].

Notation for indifinite integral.

(انظر الخلف هام)

We write: $\int 3x^2\, dx = x^3 + C$ and say in words:

"The integral of $3x^2$ with respect to x equals $x^3 + C$"

* The $\int$ sign is an elongated "s", standing for "sum", later we will see that the integral is the sum of the areas of infinitesimally (متناهية الصغر) thin rectangles.
* Σ is the symbol for "sum". it can be used for finite or infinite sums.

  $\int$ is the symbol for the sum of an finite number of infinitely small areas (or other variables).
* This "long s" notation introduced by leibniz.
* Another notation for integral is: $\boxed{F(x) = \int f(x)\, dx}$

Indifinite Integral

It's represented by $\int f(x)\,dx = F(x) + C$. where

$\int$: is the integral symbol.

$f(x)$: is the integrand (الكمية المكاملة)

x : is the integration variable.

C : is the constant of integration.

dx : is the differential of x, and indicates the variable of the fn to be integrated.

$F(x)$: is anti-derivative of $f(x)$.

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Thus $\int f(x)\,dx$, means that $f(x)$ is to be integrated with respect to x.

Remember: the variable in the function to be integrated and in the differential must be the same.

Fundamental Integration formula:

(1) $\int f'(x)\,dx = f(x) + C$ → التكامل ببساطة هو عكس التفاضل

(2) $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$ → يمكنه توزيع التكامل ع الدوال المجموعة

(3) $\int A.f(x)\,dx = A \int f(x)\,dx$, where A is any constant.

(4) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \; ; \; n \neq -1$ → تكامل ال x^n بنزود الأس 1 ونقسم ع الأس الجديد $(n+1)$.

(5) $\int x^{-1}\,dx = \int \frac{dx}{x} = \ln|x| + C.$ → وجود المعيار تحسبًا حيث أن دالة ال $\ln$ معرفة فقط للقيم $x > 0$.

(6) $\int a^x\,dx = \frac{a^x}{\ln a} + C.$ → تكامل a^x هو نفسه مقسومًا ع $\ln(a)$

(7) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C.$ → تكامل دالة ال e^{ax} هو نفسها مقسومًا ع تفاضل أسها (a)

(8) $\int \sin x\,dx = -\cos x + C.$

(9) $\int \cos x\,dx = \sin x + C.$

(10) $\int \sec^2 x\,dx = \tan x + C.$

(11) $\int \csc^2 x\,dx = -\cot x + C.$

(12) $\int \sec x . \tan x\,dx = \sec x + C.$

(13) $\int \csc x . \cot x\,dx = -\csc x + C.$

(14) $\int \sinh x . dx = \cosh x + C.$

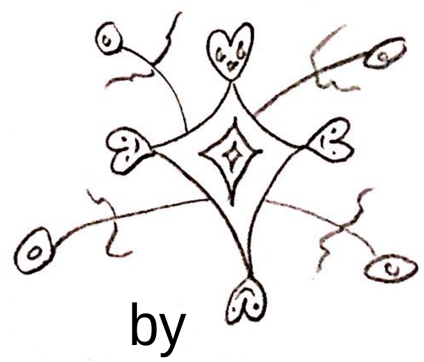
(15) $\int \cosh x\,dx = \sinh x + C.$

(16) $\int \operatorname{sech}^2 x\,dx = \tanh x + C.$

(17) $\int \operatorname{csch}^2 x\,dx = -\coth x + C.$

(18) $\int \operatorname{sech} x \tanh x . dx = -\operatorname{sech} x + C$

(19) $\int \operatorname{csch} x . \coth x\,dx = -\coth x + C.$



(20) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$

(21) $\int \frac{-dx}{\sqrt{a^2-x^2}} = \cos^{-1}\left(\frac{x}{a}\right) + C.$

(22) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$

(23) $\int \frac{-dx}{a^2+x^2} = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C.$

(24) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$

(25) $\int \frac{-dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C.$

(26) $\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C.$

(27) $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C.$

(28) $\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C$

(29) $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{-1}{a}\operatorname{sech}^{-1}\left|\frac{x}{a}\right| + C$

(30) $\int \frac{dx}{x\sqrt{a^2+x^2}} = \frac{-1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + C$

(31) $\int \frac{dx}{x^2-a^2} = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C$

Note

$$* \frac{d}{dx}\left[\sin^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2-x^2}} \quad \#$$

$$* \frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{1+\frac{x^2}{a^2}} \cdot \frac{1}{a} = \frac{a^2}{a^2+x^2} \cdot \frac{1}{a}$$

$$= \frac{a}{a^2+x^2} \quad \#$$

$$* \frac{d}{dx}\left[\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\frac{x}{a}\sqrt{\frac{x^2}{a^2}-1}} \cdot \frac{1}{a} \cdot \frac{1}{a}$$

$$= \frac{1}{x\sqrt{x^2-a^2}} \quad \#$$

Note

$$* \frac{dx}{\sqrt{x^2+a^2}} = \frac{dx}{a\sqrt{\left(\frac{x}{a}\right)^2+1}}$$

$$* \frac{dx}{a^2-x^2} = \frac{dx}{a^2\left(1-\left(\frac{x}{a}\right)^2\right)}$$

$$* \frac{dx}{x\sqrt{a^2-x^2}} = \frac{dx}{\frac{x}{a}\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

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Also:

(32) $\int \tan x\, dx = -\ln|\cos x| + C$

(33) $\int \cot x\, dx = \ln|\sin x| + C$

(34) $\int \sec x\, dx = \ln|\sec x + \tan x| + C$

(35) $\int \csc x\, dx = \ln|\csc x - \cot x| + C$

(36) $\int \cos ax\, dx = \frac{\sin ax}{a} + C$

(37) $\int \sin ax\, dx = \frac{-\cos ax}{a} + C$

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V.M.M 9/8
P.M.E 2015